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AL 101 Să se rezolve inecuaţia

$$\log_{25}\left(A_x^2\right) > \log_5\left(C_x^4\right),\ x \in \mathbb{N},\ x > 0,$$

unde A_x^2, respectiv C_x^4, înseamnă aranjamente de x luate câte 2, respectiv combinări de x luate câte 4.

$$\log_{25} A_x^2 > \log_5 C_x^4; x \in N, x \geq 4$$

$$\frac{1}{2}\log_5 A_x^2 > \log_5 C_x^4$$

$$\log_5 A_x^2 > 2 \cdot \log_5 C_x^4$$

$$\log_5 A_x^2 > \log_5 (C_x^4)^2$$

$$A_x^2 > (C_x^4)^2$$

$$x(x-1) > \left(\frac{x(x-1)(x-2)(x-3)}{1 \cdot 2 \cdot 3 \cdot 4}\right)^2$$

$$x(x-1) > \frac{x^2(x-1)^2(x-2)^2(x-3)^2}{24^2}$$

$$576 > x(x-1)(x-2)^2(x-3)^2$$

$$x(x-1)(x-2)^2(x-3)^2 < 576$$

$$x(x-1)(x-2)^2(x-3)^2 < 576$$

$$x = 4 \Rightarrow 4 \cdot 3 \cdot 2^2 \cdot 1^2 < 576(A)$$

$$x = 5 \Rightarrow 5 \cdot 4 \cdot 3^2 \cdot 2^2 < 576(F)$$

$$x = 4$$

c

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AL 102 Dacă n este numărul termenilor raţionali ai dezvoltării binomului $\left(\sqrt[3]{5}-\sqrt[5]{2}\right)^{80}$, atunci:

$$(\sqrt[3]{5}-\sqrt[5]{2})^{80} = \ldots + C_{80}^{15}(\sqrt[3]{5})^{75}(-\sqrt[5]{2})^{5} + \ldots + C_{80}^{20}(\sqrt[3]{5})^{60}(-\sqrt[5]{2})^{20} + \ldots + C_{80}^{35}(\sqrt[3]{5})^{45}(-\sqrt[5]{2})^{35} +$$

$$+ \ldots + C_{80}^{50}(\sqrt[3]{5})^{30}(-\sqrt[5]{2})^{50} + \ldots + C_{80}^{65}(\sqrt[3]{5})^{15}(-\sqrt[5]{2})^{65} + \ldots + C_{80}^{80}(-\sqrt[5]{2})^{80}$$

$n = 6$ d

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AL 103 * Determinaţi x din expresia

$$\left(x^{\log_a \sqrt{x}} + \frac{1}{x}\right)^n, \quad a > 0, \quad a \neq 1,$$

ştiind că suma coeficienţilor binomiali ai dezvoltării este 128 iar al şaselea termen al dezvoltării este egal cu $\frac{21}{a^6}$.

Pentru x=1 obținem suma coeficienților binomiali: $2^n = 128 \Rightarrow 2^n = 2^7 \Rightarrow n = 7$

$$(x^{\log_a \sqrt{x}} + \frac{1}{x})^7$$

$$k = 5$$

$$a = x^{\log_a \sqrt{x}}$$

$$b = \frac{1}{x}$$

$$n = 7$$

$$T_6 = \frac{21}{a^6}$$

$$C_7^5 (x^{\log_a \sqrt{x}})^2 \cdot \frac{1}{x^5} = \frac{21}{a^6}$$

$$\frac{7 \cdot 6}{1 \cdot 2} \cdot x^{2 \cdot \log_a \sqrt{x}} \cdot \frac{1}{x^5} = \frac{21}{a^6}$$

$$x^{\log_a x} \cdot \frac{1}{x^5} = \frac{1}{a^6}$$

$$\log_a x^{\log_a x} \cdot \frac{1}{x^5} = \log_a \frac{1}{a^6}$$

$$\log_a x^{\log_a x} + \log_a 1 - \log_a x^5 = \log_a 1 - \log_a a^6$$

$$(\log_a x)^2 - 5\log_a x + 6 = 0$$

Fie: $\log_a x = t$

$$t^2 - 5t + 6 = 0$$

$$t_1 = 2 \qquad t_2 = 3$$

$$\log_a x = 2 \qquad \log_a x = 3$$

$$x = a^2 \qquad x = a^3$$

e

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AL 104 Câţi termeni nu îl conţin pe x în dezvoltarea

$$\left(2\sqrt[3]{x}+\frac{1}{4x}\right)^n, x \neq 0,$$

ştiind că suma coeficienţilor binomiali ai dezvoltării este 256?

$$C_n^0 + C_n^1 + \ldots + C_n^n = 256$$

$$2^n = 2^8$$

$$n = 8$$

$$T_{k+1} = C_n^k a^{n-k} b^k = C_8^k (2\sqrt[3]{x})^{8-k} \left(\frac{1}{4x}\right)^k = C_8^k \cdot 2^{8-k} \cdot x^{\frac{8-k}{3}} \cdot 2^{-2k} \cdot x^{-k} = C_8^k \cdot 2^{8-3k} x^{\frac{8-k}{3}-k}$$

$$\frac{8-4k}{3} = 0 \Rightarrow k = 2$$

$$T_3 = C_8^2 \cdot 2^2 = \frac{8 \cdot 7}{1 \cdot 2} \cdot 4 = 112$$

e

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AL 105 În dezvoltarea binomului

$$\left(\sqrt{x}+\frac{1}{\sqrt[3]{x^2}}\right)^n, x>0,$$

raportul dintre coeficienţii binomiali ai termenilor al cincilea şi al treilea al dezvoltării este $\frac{7}{2}$. Dacă T_6, termenul al şaselea al dezvoltării, este de forma $\alpha \cdot x^\beta$, atunci:

$$\frac{C_n^4}{C_n^2}=\frac{7}{2}\Rightarrow \frac{\frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}}{\frac{n(n-1)}{1\cdot 2}}=\frac{7}{2}\Rightarrow \frac{(n-2)(n-3)}{12}=\frac{7}{2}\Rightarrow n^2-5n+6=42$$

$$n_{1,2}=\frac{5\pm 13}{2}\Rightarrow n_1=9;\quad n_2=-4 \text{ nu este solutie.}$$

$$T_6=C_9^5(\sqrt{x})^4\left(\frac{1}{\sqrt[3]{x^2}}\right)^5=C_9^5\cdot x^2\cdot x^{-\frac{10}{3}}=\frac{9\cdot 8\cdot 7\cdot 6}{1\cdot 2\cdot 3\cdot 4}x^{\frac{-4}{3}}=126x^{-\frac{4}{3}}$$

$$\alpha=126;\ \beta=-\frac{4}{3}$$

b

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AL 106 Să se determine coeficientul lui x^4 din dezvoltarea $(1+5x+4x^3)^{10}$.

$$(1+5x+4x^3)^{10} = C_{10}^0 + C_{10}^1(5x+4x^3) + C_{10}^2(5x+4x^3)^2 + C_{10}^3(5x+4x^3)^3 +$$

$$+ C_{10}^4(5x+4x^3)^4 + C_{10}^5(5x+4x^3)^5 + \ldots$$

$$C_{10}^2 \cdot 2 \cdot 5 \cdot 4 \cdot x^4 + C_{10}^4 \cdot 5^4 \cdot x^4$$

$$40 \cdot C_{10}^2 + 625 \cdot C_{10}^4$$

d

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AL 107 Să se determine coeficientul lui x^2 din dezvoltarea $(1+x)^3+(1+x)^4+(1+x)^5+\dots+(1+x)^{12}$.

$$T_3 = C_3^2 x^2$$

$$T_3 = C_4^2 x^2$$

....

$$T_3 = C_{12}^2 x^2$$

$$C_3^2 + C_4^2 + C_5^2 + \dots + C_{12}^2 = \frac{3}{1} + \frac{4\cdot 3}{1\cdot 2} + \frac{5\cdot 4}{1\cdot 2} + \frac{6\cdot 5}{1\cdot 2} + \frac{7\cdot 6}{1\cdot 2} + \frac{8\cdot 7}{1\cdot 2} + \frac{9\cdot 8}{1\cdot 2} + \frac{10\cdot 9}{1\cdot 2} +$$

$$+\frac{11\cdot 10}{1\cdot 2} + \frac{12\cdot 11}{1\cdot 2} = 3+6+10+15+21+28+36+45+55+66 = 285$$

d

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AL 108 * Dacă $x_1 = 9$ şi $x_{n+1} = 9^{x_n}, \forall n \in \mathbb{N}^*$, atunci ultimele două cifre ale lui x_{2013} scris în baza 10 sunt:

$$x_{2013} = x_{2012+1} = (10-1)^{2012} = C_{2012}^{0} \cdot 10^{2012} - C_{2012}^{1} \cdot 10^{2011} + C_{2012}^{2} \cdot 10^{2010} - \ldots +$$

$$+ C_{2012}^{2010} \cdot 10^{2} - C_{2012}^{2011} \cdot 10^{1} + C_{2012}^{2012} \cdot 1 = \ldots\ldots + \frac{2012 \cdot 2011}{1 \cdot 2} \cdot 100 - \frac{2012}{1} \cdot 10 + 1 =$$

$$= \ldots\ldots. + 404613200 - 20120 + 1 = \ldots\ldots.81$$

d

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AL 109 * Să se calculeze suma $S = \sum_{i=1}^{2013}\left(\sum_{j=1}^{i} j\right)$.

$$S = \sum_{i=1}^{2013}\sum_{j=1}^{i} j = \sum_{i=1}^{2013}(1+2+\ldots+i) = \sum_{i=1}^{2013}\frac{i(i+1)}{2} = \frac{1}{2}\left(\sum_{i=1}^{2013} i^2 + \sum_{i=1}^{2013} i\right) =$$

$$= \frac{1}{2}\frac{2013\cdot 2014\cdot 4027}{6} + \frac{1}{2}\frac{2013\cdot 2014}{2} = \frac{2013\cdot 2014}{4}\left(\frac{4027}{3}+1\right) =$$

$$= 671\cdot 1007\cdot 2015$$

a

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AL 110 * Fie mulţimea $A = \{0, 1, 2, 3\}$. Notăm cu N numărul de numere naturale formate din 3 elemente diferite ale mulţimii A şi cu M numărul de submulţimi de 3 elemente diferite ale mulţimii A. Să se determine N şi M.

$A = \{0;1;2;3\}$

N:

102	201	301
103	203	302
120	210	310
123	213	312
130	230	320
132	231	321

18

c

M: $\{0;1;2\}$, $\{0;1;3\}$, $\{0;2;3\}$, $\{1;2;3\}$

$$C_4^3 = C_4^1 = \frac{4}{1} = 4$$

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AL 111 * Care este probabilitatea p, respectiv q, ca să extragem un număr impar, respectiv un cub perfect (adică de forma $n^3, n \in \mathbb{N}, n > 0$), dintre numerele de la 1 la 101.

p număr impar
q cub perfect

$$p = \frac{51}{101}$$

$$q = \frac{4}{101}$$

b

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AL 112 Care este probabilitatea p, respectiv q, ca alegând una din rădăcinile ecuaţiei $(x+2)(x^2-x-1)=0$ aceasta să fie reală, respectiv întreagă.

$$(x+2)(x^2-x-1)=0$$

$$x_1=-2$$

$$x_{2,3}=\frac{1\pm\sqrt{5}}{2}$$

$$p=\frac{k}{n}=\frac{3}{3}=1$$

$$q=\frac{k}{n}=\frac{1}{3}$$

b

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AL 113 * Ştiind că după 2 reduceri succesive de 10%, respectiv 25%, urmate de o uşoară creştere cu 5% un produs costă 1134 euro, cât costa acesta înaintea primei reduceri?

x euro

$$x - x \cdot 10\% = x - 0{,}1x = 0{,}9x$$

$$0{,}9x - 0{,}9x \cdot 25\% = 0{,}9x - 0{,}9x \cdot \frac{25}{100} = 0{,}9x - x \cdot 0{,}225 = 0{,}675x$$

$$0{,}675x + 0{,}675x \cdot 5\% = 0{,}675x + 0{,}675x \cdot \frac{5}{100} = 0{,}675x + 0{,}675x \cdot 0{,}05 =$$

$$= 0{,}675x + 0{,}03375x = 0{,}70875x$$

$$0{,}70875x = 1134$$

$$x = 1600$$

c

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AL 114 Să se calculeze $|z|$ dacă

$$z=\left(\sqrt{2013+\sqrt[n]{2013}}+i\sqrt{2013-\sqrt[n]{2013}}\right)^{2n},\ n\in\mathbb{N}^*.$$

$$|z|=\left(\sqrt{(\sqrt{2013+\sqrt[n]{2013}})^2+(\sqrt{2013-\sqrt[n]{2013}})^2}\right)^{2n}=$$

$$=\left(\sqrt{2013+\sqrt[n]{2013}+2013-\sqrt[n]{2013}}\right)^{2n}=\sqrt{2\cdot 2013}^{\,2n}=$$

$$=4026^n$$

f

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AL 115 Fie mulţimea $A = \left\{ z \in \mathbb{C} \setminus \mathbb{R} : z + \dfrac{1}{z} = 1 \right\}$. Să se calculeze

$$E = z^{2013} - \frac{1}{z^{2013}}, \quad z \in A.$$

$$z + \frac{1}{z} = 1$$

$$\boxed{z^2 - z + 1 = 0}$$

$$\boxed{\begin{array}{l} z^3 + 1 = 0 \\ z^3 = -1 \end{array}}$$

$$E = z^{2013} + \frac{1}{z^{2013}} = (z^3)^{671} - \frac{1}{(z^3)^{671}} = (-1)^{671} - \frac{1}{(-1)^{671}} = -1 - \frac{1}{-1} = 0$$

f

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AL 116 Fie mulţimile

$$M=\left\{z\in\mathbb{C}:|z-2|=2 \text{ şi } \left[\frac{1}{|z-3|}\right]=1\right\},\quad P=\{|z|:z\in M\}.$$

Atunci

$$|z-2|=2$$

$$|a-2-ib|=2$$

$$\sqrt{(a-2)^2+b^2}=2$$

$$(a-2)^2+b^2=4$$

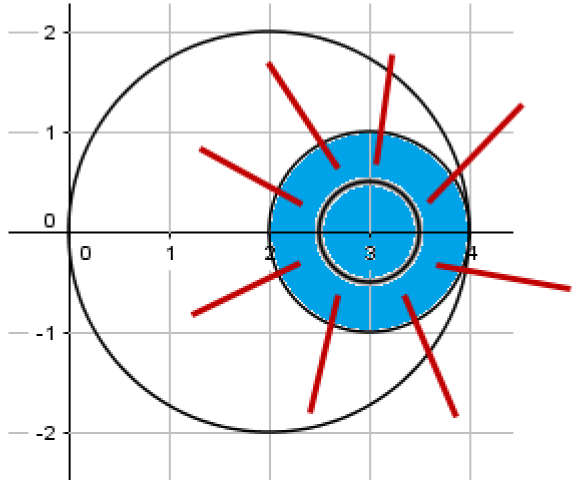
$$1\le\frac{1}{|z-3|}<2$$

$$\begin{cases}|z-3|\le 1\\ 2|z-3|>1\end{cases}$$

$$\begin{cases}|a-3+ib|\le 1\\ |a-3+ib|>\dfrac{1}{2}\end{cases}$$

$$\begin{cases}\sqrt{(a-3)^2+b^2}\le 1\\ \sqrt{(a-3)^2+b^2}>\dfrac{1}{4}\end{cases}$$

$$\begin{cases}(a-3)^2+b^2\le 1\\ (a-3)^2+b^2>\dfrac{1}{4}\end{cases}$$



Cele trei mulțimi au un singur punct comun (4;0)

$$P=\{|z|:z\in M\}=\{4\}$$

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AL 117 Fie ecuaţia

$$\left(\frac{z-i}{z+i}\right)^3+\left(\frac{z-i}{z+i}\right)^2+\frac{z-i}{z+i}+1=0.$$

Notăm cu S suma rădăcinilor ecuaţiei de mai sus. Atunci:

$$(\frac{z-i}{z+i})^3+(\frac{z-i}{z+i})^2+\frac{z-i}{z+i}+1=0$$

$$t^3+t^2+t+1=1$$

$$t^2(t+1)+(t+1)=0$$

$$(t+1)(t^2+1)=0$$

$$t_1=-1 \qquad t_2=-i \qquad t_3=-i$$

$$\frac{z-i}{z+i}=-1 \qquad \frac{z-i}{z+i}=i \qquad \frac{z-i}{z+i}=-i$$

$$z-i=-z-1 \qquad z-i=zi-1 \qquad z-i=-zi+1$$

$$2z=0 \qquad (z+1)-i(z+1)=0 \qquad (z-1)+i(z-1)=0$$

$$z_1=0 \qquad (z+1)(1-i)=0 \qquad (z-1)(1+i)=0$$

$$z_2=-1 \qquad z_2=-1$$

$$S=z_1+z_2+z_3=0$$

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AL 118 Fie mulţimea $A=\left\{z\in\mathbb{C}: z\cdot\bar{z}=2 \text{ şi } \left|\frac{2z+3}{z-3i}\right|=1\right\}$.

Dacă $S=\sum_{z\in A} z$, atunci:

$$\begin{cases} z=a+ib \\ \bar{z}=a-ib \end{cases}$$

$$\left|\frac{2z+3}{z-3i}\right|=1$$

$$\left|\frac{2a+3+2ib}{a+i(b-3)}\right|=1$$

$$|2a+3+2ib|=|a+i(b-3)|$$

$$\sqrt{(2a+3)^2+4b^2}=\sqrt{a^2+(b-3)^2}$$

$$4a^2+12a+9+4b^2=a^2+b^2-6b+9$$

$$12a+6b+6=0$$

$$2a+b+1=0$$

$$z\cdot\bar{z}=2$$

$$(a+ib)(a-ib)=2$$

$$a^2+b^2=2$$

$$\begin{cases} a^2+b^2=2 \\ 2a+b+1=0 \end{cases}$$

$$\begin{cases} a^2+(-2a-1)^2=2 \\ b=-2a-1 \end{cases}$$

$$a^2+4a^2+4a+1-2=0$$

$$5a^2+4a-1=0$$

$$5a^2+4a-1=0$$

$$a_1=\frac{1}{5}\Rightarrow b_1=-\frac{7}{5}$$

$$z_1=\frac{1}{5}-i\frac{7}{5}$$

$$a_2=-1\Rightarrow b_2=1$$

$$z_2=-1+i$$

$$S=z_1+z_2=\frac{1}{5}-i\frac{7}{5}-1+i=-\frac{4}{5}-i\frac{2}{5}$$

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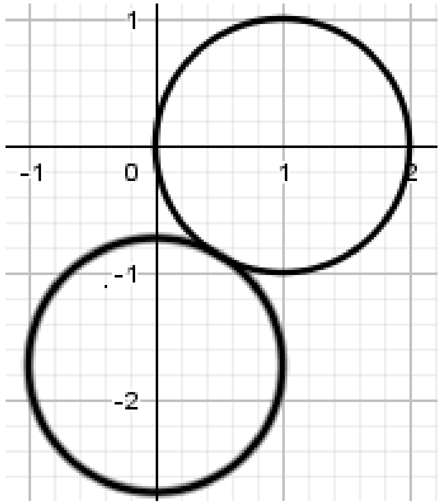
AL 119 Să se determine mulţimea $A = \left\{ z \in \mathbb{C} : |z-1| = |z+i\sqrt{3}| = 1 \right\}$.

$$z = a + ib$$

$$|z-1| = 1$$
$$|a-1+ib| = 1$$
$$\sqrt{(a-1)^2 + b^2} = 1$$
$$(a-1)^2 + b^2 = 1$$

$$|z+i\sqrt{3}| = 1$$
$$|a+i(b+\sqrt{3})| = 1$$
$$\sqrt{a^2 + (b+\sqrt{3})^2} = 1$$
$$a^2 + (b+\sqrt{3})^2 = 1$$



$$\begin{cases} (a-1)^2 + b^2 = 1 \\ a^2 + (b+\sqrt{3})^2 = 1 \end{cases} \Rightarrow \begin{cases} a^2 + b^2 - 2a = 0 \\ a^2 + b^2 + 2\sqrt{3}b = -2 \end{cases} \Rightarrow 2a + 2\sqrt{3}b = -2 \Rightarrow a + b\sqrt{3} = -1$$

$$(-1 - b\sqrt{3} - 1)^2 + b^2 = 1 \Rightarrow (-2 - b\sqrt{3})^2 + b^2 = 1 \Rightarrow 4 + 4\sqrt{3}b + 3b^2 + b^2 = 1$$

$$4b^2 + 4\sqrt{3}b + 3 = 0 \Rightarrow (2b + \sqrt{3})^2 = 0 \Rightarrow b = -\frac{\sqrt{3}}{2} \Rightarrow a = \frac{1}{2}$$

$$z = \frac{1}{2} - i\frac{\sqrt{3}}{2} \Rightarrow A = \{\frac{1}{2} - i\frac{\sqrt{3}}{2}\}$$

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AL 120 Suma modulelor rădăcinilor ecuației $z^2-(6-i)z+5-i=0$ este:

$$z^2-(6-i)z+5-i=0$$

$$\Delta=(6-i)^2-4\cdot 1\cdot(5-i)=36-12i+i^2-20+4i=16-8i+i^2=(4-i)^2$$

$$z_{1,2}=\frac{6-i\pm(4-i)}{2}\Rightarrow\begin{cases} z_1=\dfrac{10-2i}{2}=5-i \\ z_2=\dfrac{2}{2}=1\end{cases}$$

$$|5-i|=\sqrt{5^2+1^2}=\sqrt{26}$$

$$|1+0i|=\sqrt{1^2+0^2}=1$$

$$|5-i|+|1+0i|=1+\sqrt{26}$$

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AL 121 Fie mulţimea $A=\left\{z\in\mathbb{C}: z^2+z+1=0\right\}$. Să se calculeze

$$E(z)=\frac{z^{11}+z^{10}+z^9+z^8+z^7+z^6+z^3-4}{z^{2013}+2},$$

ştiind că $z\in A$.

$$z^2+z+1=0\Rightarrow z^3=1$$

$$E(z)=\frac{z^{11}+z^{10}+z^9+z^8+z^7+z^6+z^3-4}{z^{2013}+2}=$$

$$=\frac{(z^3)^3\cdot z^2+(z^3)^3\cdot z+(z^3)^3+(z^3)^2\cdot z^2+(z^3)^2\cdot z+(z^3)^2+z^3-4}{(z^3)^{671}+2}=$$

$$=\frac{z^2+z+1+z^2+z+1+1-4}{1+2}=\frac{-3}{3}=-1$$

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AL 122 Se consideră numerele complexe z_1, z_2, z_3 care îndeplinesc condiţiile $|z_1| = |z_2| = |z_3| = 1$ şi $z_1 \cdot z_2 \cdot z_3 \neq -1$. Dacă

$$B = \frac{z_1 + z_2 + z_3 - z_1 \cdot z_2 - z_1 \cdot z_3 - z_2 \cdot z_3}{1 + z_1 \cdot z_2 \cdot z_3},$$

şi $\overline{B}$ este conjugatul complex al numărului B, atunci:

$$\begin{cases} z = a + ib \\ \bar{z} = a - ib \end{cases} \Rightarrow \sqrt{a^2 + b^2} = 1 \Rightarrow a^2 + b^2 = 1 \Rightarrow z \cdot \bar{z} = 1$$

$$B = \frac{z_1 + z_2 + z_3 - z_1 z_2 - z_1 z_3 - z_2 z_3}{1 + -z_1 z_2 z_3} = \frac{\frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3} - \frac{1}{\bar{z}_1 \cdot \bar{z}_2} - \frac{1}{\bar{z}_1 \cdot \bar{z}_3} - \frac{1}{\bar{z}_2 \cdot \bar{z}_3}}{1 + \frac{1}{\bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3}} =$$

$$= \frac{\bar{z}_1 \cdot \bar{z}_2 + \bar{z}_1 \cdot \bar{z}_3 + \bar{z}_2 \cdot \bar{z}_3 - \bar{z}_1 - \bar{z}_2 - \bar{z}_3}{1 + \bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3} = -\bar{B}$$

$$B + \bar{B} = 0$$

d

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AL 123 Se consideră numerele complexe z şi w astfel încât $|z| = |w| = \sqrt{1006}$ şi $|z+w| = \sqrt{2013}$. Atunci $|z-w|$ are valoarea:

$$z = a + ib \qquad w = c + id$$

$$\begin{cases} \sqrt{a^2+b^2} = \sqrt{1006} \\ \sqrt{c^2+d^2} = \sqrt{1006} \\ \sqrt{(c+d)^2+(b+d)^2} = \sqrt{2013} \end{cases} \Rightarrow \begin{cases} a^2+b^2 = 1006 \\ c^2+d^2 = 1006 \\ (c+d)^2+(b+d)^2 = 2013 \end{cases}$$

$$\begin{cases} a^2+b^2 = 1006 \\ c^2+d^2 = 1006 \\ a^2+b^2+2ab+c^2+d^2+2cd = 2013 \end{cases} \Rightarrow \begin{cases} a^2+b^2 = 1006 \\ c^2+d^2 = 1006 \\ 2ab+2cd = 1 \end{cases}$$

$$|z-w| = \sqrt{(a-c)^2+(b-d)^2} = \sqrt{a^2-2ac+c^2+b^2-2bd+d^2} =$$

$$= \sqrt{2012-2ac-2bd} = \sqrt{2012-1} = \sqrt{2011}$$

e

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AL 124 Fie numerele complexe z şi w. Să se calculeze

$$E = \frac{1 + |z \cdot w|^2}{|\overline{z} \cdot w + 1|^2 + |z \cdot \overline{w} - 1|^2}.$$

$z = a + ib$ $\qquad$ $\overline{z} \cdot w + 1 = (a - ib)(c + id) + 1 = (ac + bd + 1) + i(ad - bc)$

$w = c + id$ $\qquad$ $z \cdot \overline{w} - 1 = (a + ib)(c - id) - 1 = (ac + bd - 1) + i(bc - ad)$

$$E = \frac{1 + (\sqrt{a^2 + b^2} \cdot \sqrt{c^2 + d^2})}{(\sqrt{(ac + bd + 1)^2 + (ad - bc)^2})^2 + (\sqrt{(ac + bd - 1)^2 + (bc - ad)^2})^2} =$$

$$= \frac{1 + (a^2 + b^2)(c^2 + d^2)}{(ac + bd + 1)^2 + (ad - bc)^2 + (ac + bd - 1)^2 + (bc - ad)^2} =$$

$$= \frac{1 + a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}{2 + 2a^2c^2 + 2a^2d^2 + 2b^2c^2 + 2b^2d^2} = \frac{1}{2}$$

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AL 125 Fie mulţimea $M = \left\{ z \in \mathbb{C} : |z| + z = 2 - 4i \right\}$. Să se determine $S = \sum_{z \in M} |z+3|^{4n}, n \in \mathbb{N}^*$.

Fie $z = a + ib \Rightarrow \sqrt{a^2 + b^2} + a + ib = 2 - 4i$

$$\begin{cases} \sqrt{a^2+b^2} + a = 2 \\ b = -4 \end{cases} \Rightarrow \sqrt{a^2+16} + a = 2 \Rightarrow \sqrt{a^2+16} = 2 - a \qquad a < 2$$

$$a^2 + 16 = 4 - 4a + a^2$$

$$4a = -12$$

$$a = \frac{-12}{4}$$

$$a = -3$$

$$z = a + ib = -3 - 4i$$

b

$$S = \sum_{z \in M} |z+3|^{4n} = |-3-4i+3|^{4n} = |-4i|^{4n} = \sqrt{0^2 + (-4)^2}^{\,4n} = 4^{4n} = 256^n$$

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AL 126 Să se determine soluţiile sistemului

$$\begin{cases} x + 2y = 1 + i, \\ 3x + iy = 2 - 3i, \quad x, y \in \mathbb{C}. \end{cases}$$

$$\begin{cases} x+2y=1+i \\ 3x+iy=2-3i \end{cases}$$

$$-6y+iy=-3-3i+2-3i$$

$$y(-6+i)=-1-6i$$

$$y=\frac{1+6i}{6-i}\cdot\frac{6+i}{6+i}=\frac{6+i+36i-36}{36+1}=\frac{37i}{37}=i$$

$$\begin{cases} x+2y=1+i \\ 3x+iy=2-3i \end{cases}$$

$$xi-6x=i-1-4+6i$$

$$x(i-6)=-5+7i$$

$$x=\frac{5-7i}{6-i}\cdot\frac{6+i}{6+i}=\frac{30+5i-42i+7}{37}=\frac{37-37i}{37}=1-i$$

c

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AL 127 Să se determine $E = \left(z^2 + \frac{1}{z^2}\right)^9$, ştiind că $z^2 - z + 1 = 0$.

$$z^2 - z + 1 = 0$$

$$z^3 + 1 = 0$$

$$z^3 = -1$$

$$E = (z^2 + \frac{1}{z^2})^9 = (\frac{z^4 + 1}{z^2})^9 = (\frac{z^3 \cdot z + 1}{z^2})^9 = (\frac{-z + 1}{z^2})^9 = (\frac{-z^2}{z^2})^9 = -1$$

b

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AL 128 Se consideră numerele complexe z_1, z_2, z_3 care îndeplinesc condiţiile $|z_1| = |z_2| = |z_3| = 1$, $z_1 + z_2 + z_3 \neq 0$ şi $z_1^2 + z_2^2 + z_3^2 = 0$. Atunci $|z_1 + z_2 + z_3|$ are valoarea:

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AL 129 Să se calculeze

$$E = \frac{i \cdot i^2 \cdot i^3 \cdot \ldots \cdot i^{2012}}{i + i^2 + i^3 + \ldots + i^{2013}}.$$

$$E = \frac{i \cdot i^2 \cdot \ldots \cdot i^{2012}}{i + i^2 + \ldots + i^{2013}} = \frac{i^{1+2+3+\ldots+2012}}{(i + i^2 + i^3 + i^4) + \ldots + (i^{2009} + i^{2010} + i^{2011} + i^{2012}) + i^{2013}} =$$

$$= \frac{i^{\frac{2012 \cdot 2013}{2}}}{i^{2013}} = \frac{i^{1006 \cdot 2013}}{i^1} = (i^{1006})^{2013-1} = i^{2 \cdot 2013 - 1} = i^{4025} = i$$

C

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AL 130 * Se consideră numerele complexe z_1, z_2, z_3 care îndeplinesc condiţiile $|z_1| = |z_2| = |z_3| = 1$ şi $z_1 + z_2 + z_3 = 1$. Să se calculeze $S = z_1^{2013} + z_2^{2013} + z_3^{2013}$.

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AL 131 Se consideră matricele $A = \begin{pmatrix} 2 & 9 \\ -7 & 4 \end{pmatrix}$ şi $B = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$.

Să se determine valoarea expresiei $B - \frac{1}{2}(A + A^t)$.

$$A^t = \begin{pmatrix} 2 & -7 \\ 9 & 4 \end{pmatrix}$$

$$A + A^t = \begin{pmatrix} 2 & 9 \\ -7 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -7 \\ 9 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 8 \end{pmatrix} = 2 \cdot \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

$$B - \frac{1}{2}(A + A^t) = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} - \frac{1}{2} \cdot 2 \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I_2$$

d

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AL 132 Se consideră matricele:

$$A = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -1 & -2 \end{pmatrix} \text{ şi } C = \begin{pmatrix} -2 & 1 & 3 \\ 4 & -5 & -6 \\ 3 & 1 & -1 \end{pmatrix}.$$

Să se calculeze $A \cdot B - C$.

$$A \cdot B - C = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 & -2 \end{pmatrix} - \begin{pmatrix} -2 & 1 & 3 \\ 4 & -5 & -6 \\ 3 & 1 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 1 & 2 \\ 6 & -2 & -4 \\ 3 & -1 & -2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & -3 \\ -4 & 5 & 6 \\ -3 & -1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 0 & -1 \\ 2 & 3 & 2 \\ 0 & -2 & -1 \end{pmatrix}$$

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AL 133 Se consideră matricele:

$$A=\begin{pmatrix} 2x & 2 \\ 2 & y^2 \end{pmatrix},\quad B=\begin{pmatrix} 3y-2 & 3x-5y \\ 3x-5y & 1+2x \end{pmatrix} \text{ și } C=\begin{pmatrix} 7 & 6 \\ 6 & 8 \end{pmatrix}.$$

Să se determine numerele reale x și y astfel încât $A+B=C$.

$$A+B=C$$

$$\begin{pmatrix} 2x & 2 \\ 2 & y^2 \end{pmatrix}+\begin{pmatrix} 3y-2 & 3x-5y \\ 3x-5y & 1+2x \end{pmatrix}=\begin{pmatrix} 7 & 6 \\ 6 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 2x+3y-2 & 3x-5y+2 \\ 3x-5y+2 & y^2+2x+1 \end{pmatrix}=\begin{pmatrix} 7 & 6 \\ 6 & 8 \end{pmatrix}$$

$$\begin{cases} 2x+3y-2=7 \\ 3x-5y+2=6 \\ y^2+2x+1=8 \end{cases}$$

$$\begin{cases} 2x+3y=9 \\ 3x-5y=4 \\ y^2+2x=7 \end{cases} \Rightarrow \begin{cases} x=3 \\ y=1 \end{cases}$$

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AL 134 Se consideră matricele:

$$A=\begin{pmatrix}-1 & x\\ x & 0\end{pmatrix},\quad B=\begin{pmatrix}y & 1\\ 2y & y\end{pmatrix}\ \text{şi}\ C=\begin{pmatrix}y & 6\\ 2x+4y & 2y\end{pmatrix}.$$

Să se determine $x, y \in \mathbb{R}$ astfel încât $xA + yB = C$.

$$x\begin{pmatrix}-1 & x\\ x & 0\end{pmatrix}+y\begin{pmatrix}y & 1\\ 2y & y\end{pmatrix}=\begin{pmatrix}y & 6\\ 2x+4y & 2y\end{pmatrix}$$

$$\begin{pmatrix}-x+y^2 & x^2+y\\ x^2+2y^2 & y^2\end{pmatrix}=\begin{pmatrix}y & 6\\ 2x+4y & 2y\end{pmatrix}$$

$$\begin{pmatrix}y^2-x & x^2+y\\ x^2+2y^2 & y^2\end{pmatrix}=\begin{pmatrix}y & 6\\ 2x+4y & 2y\end{pmatrix}$$

$$\begin{cases} y^2-x=y\\ x^2+y=6\\ x^2+2y^2=2x+4y\\ y^2=2y\end{cases}$$

$$\begin{cases} x=y^2-y\\ y=6-x^2\\ x^2-2x=-2y^2+4y\\ y(y-2)=0\end{cases}$$

$$\begin{cases} x=0\\ y=6\\ 0=0\\ y=0\end{cases}$$

nu este soluție

$$\begin{cases} x=2\\ y=6-4\\ 4-4=-4+8\\ y=2\end{cases}\Rightarrow\begin{cases} x=2\\ y=2\end{cases}$$

f

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AL 135 Se consideră matricea $A(x) = \begin{pmatrix} x+1 & x \\ 3 & x+1 \end{pmatrix}$, $x \in \mathbb{R}$.

Dacă $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, atunci $A(x)$ verifică ecuaţia:

$$A^2(x) = \begin{pmatrix} x^2+5x+1 & 2x^2+2x \\ 6x+6 & x^2+5x+1 \end{pmatrix}$$

$$\begin{pmatrix} x^2+5x+1 & 2x^2+2x \\ 6x+6 & x^2+5x+1 \end{pmatrix} - \begin{pmatrix} x^2+2x+1 & x^2+x \\ 3x+3 & x^2+2x+1 \end{pmatrix} - \begin{pmatrix} 0 & x^2+x \\ 3x+3 & 0 \end{pmatrix} - \begin{pmatrix} 3x & 0 \\ 0 & 3x \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

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AL 136 Fie matricea $A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

Să se determine $a, b, c \in \mathbb{R}$ astfel încât să aibe loc relația $A^{-1} = aA^2 + bA + cI_3$.

---

1) $\det A = -2 + 1 = -1 \neq 0 \Rightarrow \exists A^{-1}$

2) $^tA = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

3) $A^* = \begin{pmatrix} 1 & -1 & -2 \\ 1 & -2 & -2 \\ -1 & 1 & 1 \end{pmatrix}$

4) $A^{-1} = \begin{pmatrix} -1 & 1 & 2 \\ -1 & 2 & 2 \\ 1 & -1 & -1 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & -2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$A^{-1} = aA^2 + bA + cI_3$$

$$\begin{pmatrix} -1 & 1 & 2 \\ -1 & 2 & 2 \\ 1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} a+c & a+b & 2a+2b \\ -a-b & b+c & -2a \\ a+b & a & 3a+b+c \end{pmatrix}$$

$$\begin{cases} a = -1 \\ b = 2 \\ c = 0 \end{cases}$$

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AL 137 Fie matricea $A = \begin{pmatrix} 3 & 0 & -5 \\ 0 & 1 & 4 \\ -1 & 0 & 2 \end{pmatrix}$ Să se determine $a, b, c \in \mathbb{R}$ astfel încât $A^3 = aA^2 + bA + cI_3$.

$$A^2 = \begin{pmatrix} 14 & 0 & -25 \\ -4 & 1 & 12 \\ -5 & 0 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 67 & 0 & -120 \\ -24 & 1 & 48 \\ -24 & 0 & 43 \end{pmatrix} = \begin{pmatrix} 14a+3b+c & 0 & -25a-5b \\ -4a & a+b+c & 12a+4b \\ -5a-b & 0 & 9a+2b+c \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 67 & 0 & -120 \\ -24 & 1 & 48 \\ -24 & 0 & 43 \end{pmatrix}$$

$$\begin{cases} a = 6 \\ b = -6 \\ c = 1 \end{cases}$$

b

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AL 138 Fie matricele: $A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 1 & 2 \end{pmatrix}$

şi funcţia $f : \mathcal{M}_2(\mathbb{R}) \to \mathcal{M}_2(\mathbb{R})$, $f(X) = X^2 - 3X + I_2$. Să se determine matricele $B \cdot A \cdot C$ şi $f(B)$.

$$BAC = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} -2 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}\begin{pmatrix} -2 & 1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 2 \\ 4 & -2 & -4 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$$

$$f(B) = \begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix} + \begin{pmatrix} -6 & -3 \\ 9 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -4 & -2 \\ 6 & 2 \end{pmatrix} = -2B$$

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AL 139 Fie funcţia $f : \mathcal{M}_3(\mathbb{R}) \to \mathcal{M}_3(\mathbb{R}),\ f(X) = X^3 - 3X^2 + 2X + 4I_3$ şi matricea

$A = \begin{pmatrix} 0 & a & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{pmatrix}$. Să se determine valoarea parametrului real a astfel încât $f(A) = \begin{pmatrix} -4 & 24 & 8 \\ 4 & -4 & -4 \\ -4 & 8 & 8 \end{pmatrix}$.

$$A = \begin{pmatrix} 0 & a & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} a & 0 & -a \\ 1 & a & -1 \\ -1 & -a & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} a & a^2 & -a \\ a+1 & a & -a-1 \\ -a-1 & -a & a+1 \end{pmatrix}$$

$$f(A) = \begin{pmatrix} -4 & 24 & 8 \\ 4 & -4 & -4 \\ -4 & 8 & 8 \end{pmatrix}$$

$$\begin{pmatrix} a-3a+4 & a^2+2a & -a+3a \\ a+1-3+2 & a-3a+4 & -a-1+3-2 \\ -a-1+3-2 & -a+3a & a+1-3+2+4 \end{pmatrix} = \begin{pmatrix} -4 & 24 & 8 \\ 4 & -4 & -4 \\ -4 & 8 & 8 \end{pmatrix}$$

$$\begin{pmatrix} -2a+4 & a^2+2a & 2a \\ a & -2a+4 & -a \\ -a & 2a & a+4 \end{pmatrix} = \begin{pmatrix} -4 & 24 & 8 \\ 4 & -4 & -4 \\ -4 & 8 & 8 \end{pmatrix}$$

$a = 4$

f

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AL 140 Fie matricea $A = \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$. Calculaţi A^{36}.

$$A = \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix} = 2\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = 2\begin{pmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{pmatrix}$$

$$A^n = 2^n\begin{pmatrix} \cos\frac{n\pi}{6} & -\sin\frac{n\pi}{6} \\ \sin\frac{n\pi}{6} & \cos\frac{n\pi}{6} \end{pmatrix}$$

$$A^{36} = 2^{36}\begin{pmatrix} \cos 6\pi & -\sin 6\pi \\ \sin 6\pi & \cos 6\pi \end{pmatrix} = 2^{36}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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AL 141 Fie matricele: $A=\begin{pmatrix}2 & 2\\ 1 & 1\end{pmatrix}$ şi $B=A-A^t$. Calculaţi B^{2013}.

$$B=A-{}^tA=\begin{pmatrix}2 & 2\\ 1 & 1\end{pmatrix}-\begin{pmatrix}2 & 1\\ 2 & 1\end{pmatrix}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$B^2=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}=-I_2$$

$$B^3=B\cdot B^2=-B$$

$$B^4=B^2\cdot B^2=I_2$$

$$B^n=\begin{cases} B & n=4k+1\\ -I_2 & n=4k+2\\ -B & n=4k+3\\ I_2 & n=4k\end{cases}$$

$$B^{2013}=B=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

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AL 142 Fie matricea $A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$, $a \in \mathbb{R}$. Să se determine A^{2013}.

$$A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = aI_2 + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = aI_2 + B$$

$$A^n = (aI_2 + B)^n = C_n^n (aI_2)^n + C_n^{n-1}(aI_2)^{n-1} B =$$

$$= a^n I_2 + \frac{n}{1} a^{n-1} B = \begin{pmatrix} a^n & 0 \\ 0 & a^n \end{pmatrix} + \begin{pmatrix} 0 & na^{n-1} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$$

$$A^{2013} = \begin{pmatrix} a^{2013} & 2013 \cdot a^{2012} \\ 0 & a^{2013} \end{pmatrix}$$

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AL 143 * Fie matricea $A=\begin{pmatrix}1 & 1 & 2\\ 0 & 1 & 3\\ 0 & 0 & 1\end{pmatrix}$ Să se calculeze A^{100}.

$$A=\begin{pmatrix}1 & 1 & 2\\ 0 & 1 & 3\\ 0 & 0 & 1\end{pmatrix}=I_3+\begin{pmatrix}0 & 1 & 2\\ 0 & 0 & 3\\ 0 & 0 & 0\end{pmatrix}=I_3+B$$

$$B^2=\begin{pmatrix}0 & 0 & 3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$B^2=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$A^n=(B+I_3)^n=C_n^2\cdot B^2+C_n^1\cdot B+C_n^0 I_n=$$

$$=\frac{n(n-1)}{2}\begin{pmatrix}0 & 0 & 3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}+\frac{n}{1}\begin{pmatrix}0 & 1 & 2\\ 0 & 0 & 3\\ 0 & 0 & 0\end{pmatrix}+\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=$$

$$=\begin{pmatrix}1 & n & \frac{n(3n+1)}{2}\\ 0 & 1 & 3n\\ 0 & 0 & 1\end{pmatrix}$$

$$A^{100}=\begin{pmatrix}1 & 100 & \frac{100(3\cdot 100+1)}{2}\\ 0 & 1 & 3\cdot 100\\ 0 & 0 & 1\end{pmatrix}$$

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AL 144 * Să se calculeze A^{2013}, unde $A = \begin{pmatrix} 0 & e^{x} & e^{-x} \\ e^{-x} & 0 & 0 \\ e^{x} & 0 & 0 \end{pmatrix}$, $x \in \mathbb{R}$.

$$A=\begin{pmatrix} 0 & e^{x} & e^{-x} \\ e^{-x} & 0 & 0 \\ e^{x} & 0 & 0 \end{pmatrix}$$

$$A^2=\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & e^{-2x} \\ 0 & e^{2x} & 1 \end{pmatrix}$$

$$A=\begin{pmatrix} 0 & 2e^{x} & 2e^{-x} \\ 2e^{-x} & 0 & 0 \\ 2e^{x} & 0 & 0 \end{pmatrix}=2A$$

$$A^n=\begin{cases} 2^{\frac{n}{2}-1}A^2 & n=2k \\ 2^{\frac{n-1}{2}} & n=2k+1 \end{cases}$$

$$A^{2013}=2^{\frac{2013-1}{2}}A=2^{1006}A$$

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AL 145 * Se consideră H mulţimea matricelor de forma $X(a) = \begin{pmatrix} 1+4a & 6a \\ -2a & 1-3a \end{pmatrix} \in \mathcal{M}_2(\mathbb{R})$.

Să se studieze dacă $X(a)X(b) = X(a+b+ab)$, pentru orice $X(a), X(b) \in H$
şi să se calculeze $[X(1)]^n$, $n \in \mathbb{N}^*$.

$$X(a)\cdot X(b) = \begin{pmatrix} 1+4a & 6a \\ -2a & 1-2a \end{pmatrix} \cdot \begin{pmatrix} 1+4b & 6b \\ -2b & 1-2b \end{pmatrix} = \begin{pmatrix} 1+4b+4a+16ab-12ab & 6b+24ab+6a-18ab \\ -2a-8ab-2b+6ab & -12ab+1-3b-3a+9ab \end{pmatrix} =$$

$$= \begin{pmatrix} 1+4a+4b+4ab & 6a+6b+6ab \\ -2a-2b-2ab & 1-3a-3b-3ab \end{pmatrix} = X(a+b+ab)$$

$$X(1) = \begin{pmatrix} 1 & 5 \\ -2 & -2 \end{pmatrix}$$

$[X(1)]^2 = X(3)$

$[X(1)]^3 = X(7)$

....

b

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AL 146 Fie matricele: $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 16 \\ -3 & -10 & -2 \\ 8 & 0 & 0 \end{pmatrix} \in \mathcal{M}_3(\mathbb{R})$

şi $C = A^3 - 80B^{-1}$. Să se calculeze C^n, $n \in \mathbb{N}^*$.

1) Calculăm valoarea determinantului:

$$\det B = 1280$$

2) Transpusa matricei B:

$$^tB = \begin{pmatrix} 0 & -3 & 8 \\ 0 & -10 & 0 \\ 16 & -2 & 0 \end{pmatrix}$$

3) Adjuncta matricei B:

$$B^* = \begin{pmatrix} 0 & 0 & 160 \\ -16 & -128 & -48 \\ 80 & 0 & 0 \end{pmatrix}$$

4) Inversa matricei B:

$$B^{-1} = \frac{1}{\det B}\begin{pmatrix} 0 & 0 & 160 \\ -16 & -128 & -48 \\ 80 & 0 & 0 \end{pmatrix} = \frac{1}{1280}\begin{pmatrix} 0 & 0 & 160 \\ -16 & -128 & -48 \\ 80 & 0 & 0 \end{pmatrix} = \frac{1}{80}\begin{pmatrix} 0 & 0 & 10 \\ -1 & -8 & -3 \\ 5 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 7 & 0 & 10 \\ -1 & -1 & -3 \\ 5 & 0 & 7 \end{pmatrix}$$

$$C = A^3 - 80 \cdot B^{-1} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} \Rightarrow C^n = \begin{pmatrix} 7^n & 0 & 0 \\ 0 & 7^n & 0 \\ 0 & 0 & 7^n \end{pmatrix}$$

a

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AL 147 În $\mathcal{M}_2(\mathbb{R})$ se consideră matricele: $A=\begin{pmatrix} 3 & 1 \\ 4 & -1 \end{pmatrix}$ şi $B=\begin{pmatrix} 8 & -9 \\ -1 & 9 \end{pmatrix}$

Să se determine matricele X şi Y pentru care $\begin{cases} 2X-Y=A \\ 3X+2Y=B. \end{cases}$

$$\begin{cases} 2X-Y=A \\ 3X+2Y=B \end{cases} \Rightarrow \begin{cases} 4X-2Y=2A \\ 3X+2Y=B \end{cases} \Rightarrow 7X=2A+B \Rightarrow X=\frac{1}{7}\begin{pmatrix} 14 & -7 \\ 7 & 7 \end{pmatrix}=\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{cases} 2X-Y=A \\ 3X+2Y=B \end{cases} \Rightarrow \begin{cases} 6X-3Y=3A \\ -6X-4Y=-2B \end{cases} \Rightarrow -7Y=3A-2B \Rightarrow Y=-\frac{1}{7}\begin{pmatrix} -7 & 21 \\ 14 & -21 \end{pmatrix}=\begin{pmatrix} 1 & -3 \\ -2 & 3 \end{pmatrix}$$

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AL 148 Fie sistemul
$$\begin{cases} 2A+B=\begin{pmatrix} 4 & 7 & -4 \\ 1 & 4 & 3 \\ 9 & 3 & 10 \end{pmatrix} \\ A-3B=\begin{pmatrix} -5 & 0 & 5 \\ -3 & 2 & -2 \\ 8 & -2 & -9 \end{pmatrix} \end{cases}$$

Să se calculeze Trace (A) + det (B), unde Trace(A) este suma elementelor de pe diagonala principală a matricei A.

$$\begin{cases} 2A+B=\begin{pmatrix} 4 & 7 & -4 \\ 1 & 4 & 3 \\ 9 & 3 & 10 \end{pmatrix} \\ A-3B=\begin{pmatrix} -5 & 0 & 5 \\ -3 & 2 & -2 \\ 8 & -2 & -9 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 6A+3B=\begin{pmatrix} 12 & 21 & -12 \\ 3 & 12 & 9 \\ 27 & 9 & 30 \end{pmatrix} \\ A-3B=\begin{pmatrix} -5 & 0 & 5 \\ -3 & 2 & -2 \\ 8 & -2 & -9 \end{pmatrix} \end{cases} \Rightarrow 7A=\begin{pmatrix} 7 & 21 & -7 \\ 0 & 14 & 7 \\ 35 & 7 & 21 \end{pmatrix} \Rightarrow A=\begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix} \quad Trace(A)=6$$

$$\begin{cases} 2A+B=\begin{pmatrix} 4 & 7 & -4 \\ 1 & 4 & 3 \\ 9 & 3 & 10 \end{pmatrix} \\ A-3B=\begin{pmatrix} -5 & 0 & 5 \\ -3 & 2 & -2 \\ 8 & -2 & -9 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 2A+B=\begin{pmatrix} 4 & 7 & -4 \\ 1 & 4 & 3 \\ 9 & 3 & 10 \end{pmatrix} \\ -2A+6B=\begin{pmatrix} 10 & 0 & -10 \\ 6 & -4 & 4 \\ -16 & 4 & 18 \end{pmatrix} \end{cases} \Rightarrow 7B=\begin{pmatrix} 14 & 7 & -14 \\ 7 & 0 & 7 \\ -7 & 7 & 28 \end{pmatrix} \Rightarrow B=\begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & 1 \\ -1 & 1 & 4 \end{pmatrix} \det(B)=-2-1-2-4=-9$$

$$Trace(A)+\det(B)=6-9=-3$$

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AL 149 Să se determine matricea X care verifică relaţia

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \cdot X = \begin{pmatrix} -4 & 2 & -6 \\ 6 & -3 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot X = \begin{pmatrix} -4 & 2 & -6 \\ 6 & -3 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} a & b & c \end{pmatrix} = \begin{pmatrix} -4 & 2 & -6 \\ 6 & -3 & 9 \end{pmatrix}$$

$$\begin{pmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \end{pmatrix} = \begin{pmatrix} -4 & 2 & -6 \\ 6 & -3 & 9 \end{pmatrix}$$

$$\begin{cases} a = 2 \\ b = -1 \Rightarrow X = \begin{pmatrix} 2 & -1 & 3 \end{pmatrix} \\ c = 3 \end{cases}$$

d

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AL 150 Să se rezolve ecuaţia matriceală $X \cdot \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 3 \\ 3 & -2 & 2 \end{pmatrix}$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\det A = -3$$

$$^{t}A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix}$$

$$A^{*} = \begin{pmatrix} -1 & 2 & -3 \\ 2 & -1 & -3 \\ -1 & -1 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & 3 \\ 1 & 1 & -3 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -1 & 3 \\ 3 & -2 & 2 \end{pmatrix} \cdot \frac{1}{3} \cdot \begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & 3 \\ 1 & 1 & -3 \end{pmatrix} =$$

$$= \frac{1}{3}\begin{pmatrix} 7 & -2 & -6 \\ 9 & -6 & -3 \end{pmatrix} = \begin{pmatrix} \frac{7}{3} & -\frac{2}{3} & -2 \\ 3 & -2 & -1 \end{pmatrix}$$

e